

# The 47<sup>th</sup> International Collegiate Programming Contest Asia Hong Kong Regional Contest

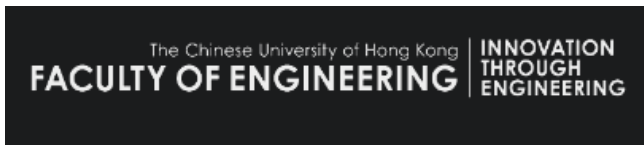
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*Do not open before the contest has started.*

## Problem A. TreeScript

Input file:            **standard input**  
Output file:           **standard output**  
Time limit:            1 second  
Memory limit:         1024 megabytes

TreeScript is a programming language developed for maintaining tree structures. In this problem, we will learn how to create rooted trees in TreeScript.

In TreeScript, all tree nodes are stored in memory. Each tree node has a number and the address of its parent node, and both are immutable, so they have to be determined when creating the node. In particular, the address of the root node's parent node is empty.

In order to access these nodes, the address of a node can be stored in a register. If there are  $m$  registers, the registers can be written as  $r[0], r[1], \dots, r[m - 1]$ .

Now let's learn the node creation statement:

$$r[i] = \text{create}(r[j], k);$$

where  $k$  is the node number,  $i$  and  $j$  are the indices of the registers, where  $0 \leq i, j < m$  and  $i = j$  is possible. The effect of this statement is that a node numbered  $k$  is created, whose parent address is stored in  $r[j]$ , and then the new node's address is stored in  $r[i]$ . Once each node has been created correctly, you do not need to store the address of any more nodes; they will automatically execute the pre-defined instructions. For reasons of space, we will learn about them later.

To check your learning, you need to create a rooted tree of size  $n$ . At first, the system will automatically create the root node for you and store it in  $r[0]$ . So you only need to execute  $n - 1$  additional creation instructions to create the tree.

As you know, registers are very expensive, so you need to find the minimum value of the registers you need.

### Input

There are multiple test cases.

The first line of the input contains one integer  $T$  ( $1 \leq T \leq 10^5$ ) - the number of test cases.

For each test case,

The first line contains one integer  $n$  ( $2 \leq n \leq 2 \times 10^5$ ) - the size of the tree.

The second line contains  $n$  integers  $p_1, p_2, \dots, p_n$ , where node  $p_i$  is the parent node of node  $i$  and  $1 \leq p_i < i$ . Specially,  $p_1 = 0$  and it means 1 is the root of the tree.

The sum of  $n$  over all test cases does not exceed  $2 \times 10^5$ .

### Output

For each test case, output the answer in one line.

### Example

standard input	standard output
2	1
3	2
0 1 2	
7	
0 1 2 2 1 4 1	

## Problem B. Big Picture

Input file:            **standard input**  
Output file:           **standard output**  
Time limit:            1 second  
Memory limit:         1024 megabytes

Grammy has a big picture with  $n + 1$  rows and  $m + 1$  columns. Rows are numbered from 1 to  $n + 1$  and columns are numbered from 1 to  $m + 1$ .

Grammy decides to color this picture in a special way. For the  $i$ -th row, Grammy will color the leftmost  $j$  ( $1 \leq j \leq m$ ) cells black with probability  $p_{i,j}$ . For the  $j$ -th column, Grammy will color the topmost  $i$  ( $1 \leq i \leq n$ ) cells black with probability  $q_{i,j}$ . Operations are independent, and a cell could be colored more than once.

Let us define the beauty value as the number of maximal orthogonally connected regions of the same color. Before Grammy finishes her coloring, she wants to know the expected number of regions on the picture. Please calculate the expected beauty value of the picture for her.

Two cells  $x$  and  $y$  are in the same orthogonally connected region if and only if they satisfy the following constraints:

- They have the same color.
- $x$  shares an edge with  $y$  or  $x$  shares an edge with some cell  $z$  while  $y$  and  $z$  are in the same orthogonally connected region.

### Input

The first line contains two integers  $n, m$  ( $1 \leq n, m \leq 1000$ ), denoting the size of the picture.

Each of the next  $n$  lines contains  $m$  integers  $p_{i,j}$ , denoting the probability of painting the leftmost  $j$  cells of the  $i$ -th row black, modulo 998 244 353. It is guaranteed that the sum of the probability of each row is 1.

Each of the next  $n$  lines contains  $m$  integers  $q_{i,j}$ , denoting the probability of painting the topmost  $i$  cells of  $j$ -th column black, modulo 998 244 353. It is guaranteed that the sum of the probability of each column is 1.

### Output

Output a single integer, denoting the expected beauty value of the picture, modulo 998 244 353.

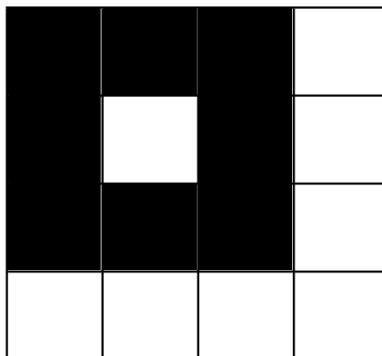
It can be shown that the answer can be expressed as an irreducible fraction  $\frac{x}{y}$ , where  $x$  and  $y$  are integers and  $y \not\equiv 0 \pmod{998\,244\,353}$ . Output the integer equal to  $x \cdot y^{-1} \pmod{998\,244\,353}$ . In other words, output such an integer  $a$  that  $0 \leq a < 998\,244\,353$  and  $a \cdot y \equiv x \pmod{998\,244\,353}$ .

## Examples

standard input	standard output
3 3 0 0 1 1 0 0 0 0 1 0 1 0 0 0 0 1 0 1	3
2 2 499122177 499122177 499122177 499122177 499122177 499122177 499122177 499122177	2
3 3 332748118 332748118 332748118 332748118 332748118 332748118 332748118 332748118 332748118 332748118 332748118 332748118 332748118 332748118 332748118 332748118 332748118 332748118	308100111

## Note

There is only one possible picture in the first example, which is shown as follows. There are 3 maximal orthogonally connected regions in the picture, so the beauty value of the picture is 3.



## Problem C. Painting Grid

Input file:            **standard input**  
Output file:           **standard output**  
Time limit:            1 second  
Memory limit:         1024 megabytes

Grammy has an  $n \times m$  wall covered by squares. Each small square on the wall is of unit size and should be painted into one color completely. She wants to color the wall into black and white. Grammy likes the concept of diversity, so she decided to make each row look different from all previous rows and also make each column look different from all previous columns. As she was about to paint, she found her paint was just enough - half of white paint and half of black paint, both with an amount to paint exactly  $\frac{nm}{2}$  unit area. Please help Grammy to satisfy her diversity condition using limited paint.

### Input

The input contains multiple test cases.

The first line contains a single integer  $T$  ( $1 \leq T \leq 2000$ ), denoting the number of test cases.

For each test case:

The only line contains two integers  $n, m$  ( $1 \leq n, m \leq 1000$ ). It is guaranteed that the sum of  $nm$  does not exceed  $10^6$ .

### Output

For each test case, if no solution exists, output “NO”. Otherwise, output “YES” followed by  $n$  lines. Each line should contain  $m$  characters. 0 denotes a white square and 1 denotes a black square in the solution.

### Example

standard input	standard output
5	NO
1 1	YES
2 2	10
2 4	01
4 4	YES
5 10	1100
	0110
	YES
	1100
	0110
	0000
	1111
	YES
	1111100000
	0101010101
	0011011001
	0000111110
	1111000001

## Problem D. Shortest Path Query

Input file:            standard input  
Output file:           standard output  
Time limit:            2 seconds  
Memory limit:         1024 megabytes

You will be given a directed acyclic graph with  $n$  vertices, labeled by  $1, 2, \dots, n$ . There are  $m$  edges in the graph, each edge is either black or white. It is guaranteed that you can reach every vertex from the 1-st vertex.

You will be given  $q$  queries. In the  $i$ -th query, you will be given three integers  $a_i, b_i$  and  $x_i$ . You need to report the length of the shortest path from the 1-st vertex to the  $x_i$ -th vertex if we regard the length of each black edge as  $a_i$  and regard the length of each white edge as  $b_i$ .

### Input

The first line of the input contains two integers  $n$  and  $m$  ( $1 \leq n \leq 50\,000, 1 \leq m \leq 100\,000$ ), denoting the number of vertices and the number of directed edges.

In the next  $m$  lines, the  $i$ -th line contains three integers  $u_i, v_i$  and  $c_i$  ( $1 \leq u_i < v_i \leq n, v_i - u_i \leq 1\,000, 0 \leq c_i \leq 1$ ), describing a directed edge from the  $u_i$ -th vertex to the  $v_i$ -th vertex. When  $c_i = 0$ , its color is black, and when  $c_i = 1$ , its color is white.

The next line contains a single integer  $q$  ( $1 \leq q \leq 50\,000$ ), denoting the number of queries.

Each of the next  $q$  lines contains three integers  $a_i, b_i$  and  $x_i$  ( $1 \leq a_i, b_i \leq 10\,000, 1 \leq x_i \leq n$ ), denoting a query.

It is guaranteed that you can reach every vertex from the 1-st vertex.

### Output

For each query, print a single line containing an integer, denoting the length of the shortest path.

### Example

standard input	standard output
4 4	3
1 2 0	4
1 3 1	4
2 4 0	
3 4 1	
3	
3 5 2	
3 2 4	
2 3 4	

## Problem E. Goose, goose, DUCK?

Input file:            standard input  
Output file:           standard output  
Time limit:           4 seconds  
Memory limit:         1024 megabytes

In the game ‘Goose Goose Duck’, the geese’s target is to complete the tasks and stay alive. However the saboteurs, ducks, will try to stop the geese from completing the tasks by killing them.

Now consider a game consisting of  $n$  geese and  $k$  ducks. The geese are numbered from 1 to  $n$ , and the  $i$ -th goose can complete the task numbered  $a_i$ . The geese decided to dispatch an interval of geese to complete the task, which means they will choose two integers  $l, r$ , satisfying  $1 \leq l \leq r \leq n$ , and all geese numbered  $i$  which satisfy that  $l \leq i \leq r$  will go to complete their task. Such a decision is called a plan, two plans are considered different if and only if the interval is different.

The ducks, will crouch at a task location, and kill all the geese trying to complete this task. They can not choose a task location which more than  $k$  geese will come, because they can’t kill them all and there will be witness, they also can not choose a task location which less than  $k$  geese will come, because they will kill their teammates by mistake. In other words, they can only choose a task location where exactly  $k$  geese will come.

A plan is said to be dangerous if and only if there exist a task location that the ducks can ambush. Please help the geese to count how many plans are **not dangerous** for the geese. Please notice that the geese do not have to complete all the tasks with the plan.

### Input

The first line contains two integers  $n, k$  ( $1 \leq n, k \leq 10^6$ ).

The second line contains  $n$  integers, the  $i$ -th integer  $a_i$  ( $1 \leq a_i \leq 10^6$ ) denotes the task number of the goose numbered  $i$ .

### Output

Output one line containing one integer, denoting the answer.

### Examples

standard input	standard output
6 2 1 2 2 1 3 3	10
6 1 1 2 3 4 5 6	0

## Problem F. Sum of Numbers

Input file:            standard input  
Output file:           standard output  
Time limit:           1 second  
Memory limit:         1024 megabytes

Given  $n$  digits from '1' to '9', you can add  $k$  '+' to turn it into an expression, find the minimum value of the expression.

### Input

There are multiple test cases.

The first line of the input contains one integer  $T$  ( $1 \leq T \leq 2 \times 10^4$ ) - the number of test cases.

For each test case,

The first line contains two integers  $n$  ( $2 \leq n \leq 2 \times 10^5$ ) and  $k$  ( $1 \leq k \leq 6, k < n$ ) - the number of digits and the number of '+'.  
The second line contains a string of length  $n$ , which consists of digits from '1' to '9'.  
The sum of  $n$  over all test cases does not exceed  $2 \times 10^5$ .

### Output

For each test case, output the answer in one line.

### Example

standard input	standard output
2	9696
8 1	6
45455151	
2 1	
42	



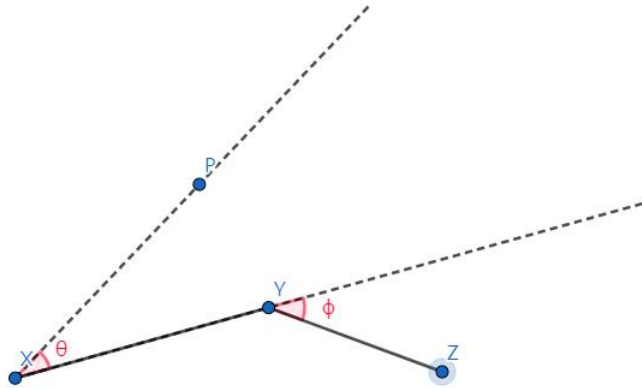
## Problem G. Paddle Star

Input file:            standard input  
Output file:           standard output  
Time limit:           1 second  
Memory limit:         1024 megabytes

Putata is fascinated with playing Zoe these days. Putata is practicing Zoe's ability, paddle star at this moment.

This ability allows Zoe to fire a star that she can redirect in flight. The damage dealt by the star is positively correlated to its flying distance. Formally, assume Zoe is currently at point  $X$  and she fires in any direction. At any moment, Zoe can change the direction of the star. If the star is at point  $Y$ , and the new direction leads to some point  $Z$ , then the track of the star will be  $X \rightarrow Y \rightarrow Z$ . If the star meets any enemy on its trace, it will immediately disappear and cause damage to the enemy.

Since Putata is not skilled enough to control this ability, the stars he fires always fit the following restrictions. The first segment  $XY$  always has length  $l_1$ , and the second segment  $YZ$  always has length  $l_2$ . Assume Zoe is facing direction  $\vec{XP}$  (which is a certain direction), and let  $\langle \vec{XP}, \vec{XY} \rangle$  be  $\theta$  degrees,  $\langle \vec{XY}, \vec{YZ} \rangle$  be  $\phi$  degrees, then  $\theta \in [-\alpha, \alpha]$  and  $\phi \in [-\beta, \beta]$ . You can refer to the following picture to help you understand.



Putata's rival, Budada, wants to know the total area that could possibly be attacked by Putata's Zoe. However, Budada is busy with math problems, so he wants you to help him solve this problem.

### Input

The first line contains one integer  $T$  ( $1 \leq T \leq 10^5$ ), the number of test cases.

For each test case output one line contains four integers  $l_1, l_2, \alpha$  and  $\beta$  ( $1 \leq l_2 \leq l_1 \leq 10^9$ ,  $0 \leq \alpha \leq 90$ ,  $0 \leq \beta < 180$ ), indicating the attributes of the ability.

### Output

Print one line for each test case, each line contains one real number, indicating the answer.

Your answer will be considered correct if its absolute or relative error does not exceed  $10^{-6}$ . Formally, let your answer be  $a$ , and the jury's answer be  $b$ . Your answer will be considered correct if  $\frac{|a-b|}{\max(1,|b|)} \leq 10^{-6}$ .

## Example

standard input	standard output
5	3.490658503988659
2 1 20 20	0.000000000000000
3 3 0 0	3367.157611906510738
20 20 90 120	1098.863278984081717
20 10 50 170	373.960489570087645
100 10 1 93	

## Problem H. Another Goose Goose Duck Problem

Input file:            standard input  
Output file:           standard output  
Time limit:            1 second  
Memory limit:         1024 megabytes

Teacher Rice likes playing the famous game ‘Goose Goose Duck’. In the game, Teacher Rice plays a duck and his goal is to kill the geese. Every time he kills a goose, he should wait  $a$  seconds for his killing skill to cool down. Since Teacher Rice’s role is the Serial Killer, the time Teacher Rice waits depends on which type of goose he kills. Because Teacher Rice is a skilled killer, he can make the waiting time  $a$  to be an arbitrary integer in  $[l, r]$ .

Teacher Rice meets a goose every  $b$  seconds. Once Teacher Rice meets a goose, he can choose to kill the goose if his killing skill is ready, otherwise the goose runs away immediately and he can not kill this goose.

Teacher Rice wants to know the minimum time he needs to kill  $k$  geese.

### Input

There are four integers in one line,  $l, r, b, k$  ( $1 \leq l \leq r \leq 10^9$ ,  $1 \leq b, k \leq 10^9$ ).

### Output

Output one integer denotes the time Teacher Rice needs to kill  $k$  geese.

### Examples

standard input	standard output
6 6 3 3	18
2 3 5 4	20

## Problem I. Range Closest Pair of Points Query

Input file:            **standard input**  
Output file:           **standard output**  
Time limit:            7 seconds  
Memory limit:         1024 megabytes

The closest pair of points problem is a well-known problem of computational geometry. In this problem, there are  $n$  points  $p_1, p_2, \dots, p_n$  in the Euclidean plane. You will be given  $q$  queries. In the  $i$ -th query, you will be given two integers  $l_i$  and  $r_i$  ( $1 \leq l_i < r_i \leq n$ ). You need to find a pair of points  $(u, v)$  such that  $l_i \leq u < v \leq r_i$  and the Euclidean distance  $\sqrt{(x_u - x_v)^2 + (y_u - y_v)^2}$  between point  $p_u$  and  $p_v$  is minimized.

### Input

The first line of the input contains two integers  $n$  and  $q$  ( $2 \leq n \leq 250\,000$ ,  $1 \leq q \leq 250\,000$ ), denoting the number of points and the number of queries.

In the next  $n$  lines, the  $i$ -th line contains two integers  $x_i$  and  $y_i$  ( $1 \leq x_i, y_i \leq 10^8$ ), describing the coordinate of  $p_i$ .

Each of the next  $q$  lines contains two integers  $l_i$  and  $r_i$  ( $1 \leq l_i < r_i \leq n$ ), denoting a query.

### Output

For each query, print a single line containing an integer, denoting the value of  $(x_u - x_v)^2 + (y_u - y_v)^2$ .

### Examples

standard input	standard output
5 5	2
2 4	8
1 1	8
3 3	2
5 1	2
4 2	
1 5	
2 3	
2 4	
3 5	
1 3	
2 1	0
1 1	
1 1	
1 2	

## Problem J. Dice Game

Input file:            **standard input**  
Output file:           **standard output**  
Time limit:            4 seconds  
Memory limit:         1024 megabytes

Putata and Budada are playing an interesting game. They play this game with a dice having  $n$  faces. Every integer between 0 and  $n - 1$  are written on exactly one face, and when they roll this dice, each side will face up with equal probability. In other words, rolling the dice will result in a uniform random integer between 0 and  $n - 1$  with equal probability.

The game has two rounds. In the first round, the following happens:

- Putata will roll the dice and get an integer as the result, say  $x$ .

In the second round, Budada can choose to do one of the following things:

- End the game, and the score of the game will be  $x$ .
- Roll the dice again, let the result be  $y$ , and the game will end, the score of the game will be  $x \oplus y$ . Here  $\oplus$  denotes binary exclusive-or operation.

Putata and Budada wants to maximize the score of the game, and they are clever so that they will always make the best choice. Please write a program to calculate for some given  $n$ , the expectation of the score of the game.

It can be shown that the answer can be expressed as an irreducible fraction  $\frac{x}{y}$ , where  $x$  and  $y$  are integers and  $y \not\equiv 0 \pmod{998\,244\,353}$ . Output the integer equal to  $x \cdot y^{-1} \pmod{998\,244\,353}$ . In other words, output such an integer  $a$  that  $0 \leq a < 998\,244\,353$  and  $a \cdot y \equiv x \pmod{998\,244\,353}$ .

### Input

The input contains several test cases. The first line contains an integer  $T$  ( $1 \leq T \leq 10^4$ ).

For the following  $T$  lines, each line contains an integer  $n$  ( $1 \leq n \leq 998\,244\,352$ ), denoting one question.

### Output

Output  $T$  lines, each line denotes the answer for one test case.

### Example

standard input	standard output
4	0
1	249561089
2	776412276
3	2
4	

## Problem K. Maximum GCD

Input file:            standard input  
Output file:           standard output  
Time limit:            1 second  
Memory limit:         1024 megabytes

Grammy has a array of length  $n$ . She recently learned about the concept of greatest common divisor(GCD). Recall that the GCD of an array is the maximum integer  $d$  such that every element in the array is divisible by  $d$ . Grammy thinks that the GCD of an array should be as large as possible so that the array can be beautiful.

You want to help Grammy to make her array beautiful, so you decided to do some(possibly, zero) modulo operations on each of the elements in the array. In other words, you can choose a number  $a_i$  ( $1 \leq i \leq n$ ) in the array and choose another integer  $x$  and replace  $a_i$  with  $(a_i \bmod x)$  in each operation. Since Grammy does not want 0 to appear in her array, you cannot change  $a_i$  into 0 by doing the modulo operation.

Now, your task is to calculate the maximum GCD of the array after several(possibly, zero) modulo operations.

### Input

The first line contains a single integer  $n$  ( $1 \leq n \leq 10^5$ ), denoting the number of elements in the array.

The second line contains  $n$  positive integers  $a_i$  ( $1 \leq a_i \leq 10^9$ ), denoting the initial elements of Grammy's array.

### Output

Output a single integer, denoting the maximum GCD of the array after any number of modulo operations.

### Example

standard input	standard output
3 3 10 7	3

## Problem L. Permutation Compression

Input file:            **standard input**  
Output file:           **standard output**  
Time limit:            1 second  
Memory limit:         1024 megabytes

Grammy has a permutation of length  $n$ . She wants to delete some useless elements in the permutation, so she decided to use some magic tool to delete them. There are  $k$  magic tools, the  $i$ -th of them can delete the maximum element of an interval of length exactly  $l_i$ . Each magic tool can be used at most once.

Before using the tool, Grammy shows you her blueprint of the array after deletion. The new array consists of exactly  $m$  distinct elements from 1 to  $n$ . Please help Grammy to determine whether it is possible to delete the elements by using the magic tool.

### Input

There are multiple test cases. The first line contains an integer  $T$  ( $1 \leq T \leq 10^5$ ), denoting the number of test cases.

For each testcase:

The first line contains 3 integers  $n, m, k$  ( $1 \leq m \leq n \leq 2 \times 10^5, 1 \leq k \leq 2 \times 10^5$ ), denoting the length of the permutation, the length of the compressed array, and the parameter of the magic tool.

The second line contains  $n$  distinct integers  $a_i$  ( $1 \leq a_i \leq n$ ), denoting the initial permutation. It is guaranteed that the elements are distinct.

The third line contains  $m$  distinct integers  $b_i$  ( $1 \leq b_i \leq n$ ), denoting the array after compression. It is guaranteed that the elements are distinct.

The fourth line contains  $k$  integers  $l_i$  ( $1 \leq l_i \leq n$ ), denoting the magic tools.

It is guaranteed that  $\sum n \leq 2 \times 10^5$  and  $\sum k \leq 2 \times 10^5$ .

### Output

For each testcase, output “YES” or “NO” in a separate line, denoting the answer to the problem.

### Example

standard input	standard output
3	YES
5 2 3	YES
5 1 3 2 4	NO
5 2	
1 2 4	
5 5 5	
1 2 3 4 5	
1 2 3 4 5	
1 2 3 4 5	
3 2 2	
3 1 2	
3 2	
2 3	